# Analysis of the Effect of Electrical Torque on Quadcopter Behaviour for Optimal Performance using MATLAB

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**Abstract**— This paper focus on analyzing the behaviour of a Quadcopter from its general principle of operation considering the effect of electrical torque when current flows through its major control unit, motors. Detail analysis of the Brushless DC motor used on the quadcopter was carried out using Emax MT1806-2280Kv motor as guide and its transfer function derived considering the relationship between the current flowing and the applied Voltage. Matlab was used for the detail analysis of the time and Frequency-Domain response of this motor. A Proportional Integral Derivative(PID) controller was also used for the performance evaluation of the motor which was analyzed using MATLAB and Ziegler Nicholas algorithm for tuning. Block diagram depicting the arrangement of the four motors on the quadcopter illustrating effect of torque on the quadcopter was modelled and performance evaluation was carried out under different cases of varying input voltage to the quadcopter to ascertain the stability level of the system at those levels.

**Index Terms**— Torque, Quadcopter, Matlab, Optimal Performance, PID, Ziegler Nicholas

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## 1 INTRODUCTION

The most used type of Unmanned Aerial Vehicle known as Quadcopter has gained its popularity among hobbyist and Researchers in the field of Aerodynamics due to its special features which includes its simplicity in design and its ability to be controlled easily by adjusting the speed of the Brushless DC motor based on the Pulse Width modulated Signal sent to the it. The four motor arranged at its corner bring about the name Quad-copter. Similar to the helicopter, it has propeller attached to each of its motors which are directed upwards and by rapidly spinning these rotors pushes air downwards, thus creating a thrust force keeping it aloft [1]. The arrangement of the four motors are such that they are equally spaced at the corners of the squared body. The motors are placed in a square formation with equal distance from the center of body mass of the quadcopter. The rotation of these motors is such that two adjacent motor moves in opposite direction. This is done to enable the thrust created by such rotations be concentrated at the center of the quadcopter opposing external forces trying to pull it down so as to ensure its lifting.
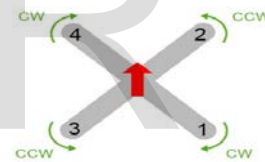


Fig 1. Rotating direction of the motors arranged on the quadcopter corner

The simplicity of this quadcopter design come at the cost of its stability. Improving its capability and efficiency by adjusting the system stability to be more stable requires detail consideration and analysis of those factors that constitute this Problem of Instability. As the speed of the motors attached on the frame of the quadcopter is being altered to produce Thrust which enables the movement of the quadcopter, it also produces an electrical torque which has a direct positive or negative effect on the stability of the system. The effect caused by this torque if not properly combatted can cause the whole system to be Unstable and can leads to Unexpected crash of the system during Flight.

This paper focus on analyzing the effect of electrical torque on the physical behaviour of Quadcopter for optimal control.

Detail theoretical studies was carried out and executed followed by a simple experimental setup to visualize this study.

## 2 Mathematical Model

The Brushless DC motors (Emax MT1806-2280Kv) used on the quadcopter for analysis in this research are of the same type and specification. Thrust produced by all the motors sum up algebraically to enable the movement of the whole quadcopter. Torque is also generated as current flow through the motor. Based on this principle of the effect of torque on its movement, the mathematical model of the quadcopter can be derived, its transfer function determined which can be further analyzed using various control system tools.

The Model of a quadcopter is a nonlinear Multivariable system with 6 Degree of Freedom (DoF) and only 4 actuators which serve as its control. In this Research, the mathematical model of a full quadcopter will be derived based on the effect of torque on its actuators relative to one another. The Aerodynamics forces and gyroscopic effects are neglected in this project because it is intended that

i. The quadcopter is restricted from flying in air as it is to be attached to a pivot of mass heavy enough to withstand its maximum pull due to thrust.

ii. The quadcopter is free of any external forces except that due to friction between the quadcopter and the pivot which it is attached to.

For similar motors used on the quadcopter, analysis of one of the motor is done and its relation to others was used to model the Quadcopter.

### 2.1   Model of The Effect of Torque on Brushless DC Motor

A Brushless DC motor is controlled by the flight controller by sending a pulse width modulated signal to it via an electronic speed controller which based on the PWM signal control the sequence of switching of the motor voltage. The motor is said to be an armature controlled DC motor. The Schematic representing the motor is given below:
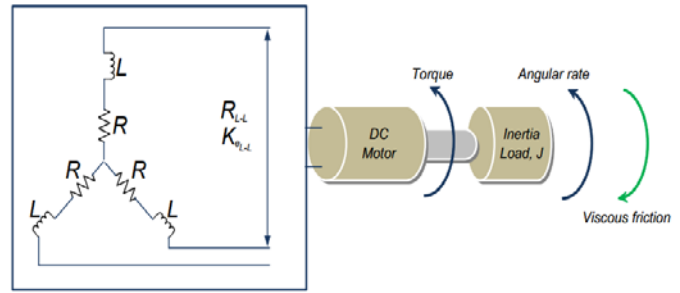


Fig 2 Brushless Motor Schematic [2]

The electrical analysis of the motor can be studied using the Kirchhoff Voltage Law equation. The Input Voltage Vs is given by Saeed [3] as

$$V_s = Ri_a + L\frac{di_a}{dt} + e \qquad (1)$$

Where

R is the motor resistance in Ohms ($\Omega$), L is motor inductance in Henry (H), e is back-EMF in Volt (V) and $i_a$ is the armature current in Ampere (A).

The mechanical properties of the dc motor are represented as

$$T_e = K_f\omega_m + J\frac{d\omega_m}{dt} \qquad (2)$$

Where

$T_e$ is the electrical torque which is a result of armature current in (N-m), $K_f$ is the friction constant, $J$ is the Motor inertia in (kg-m²), $\omega$ in (rad/s) is the angular velocity.

The back EMF $e$ is related with the angular speed of the rotor, the torque produced by the motor is a function of the supplied current flowing through the armature.

$$e = K_b\frac{d\theta}{dt} \qquad (3)$$

$$T_e = K_t i_a \qquad (4)$$

$$\omega_m = \frac{d\theta}{dt} \qquad (5)$$

Where

$K_b$ is the equivalent coefficient of friction and $K_t$ is the torque constant. Assuming a variable supply voltage to the motor which can be varied with time. then its equation can be split and written as

$$Vs = Ku(t) \qquad (6)$$

Where

K is voltage constant which is used in adjusting the input voltage and u(t) is the input voltage in Volt (V). the Electrical

and Mechanical equation of the system can be written as

$$Ku(t) = Ri + L\frac{di}{dt} + K_b\frac{d\theta}{dt} \qquad (7)$$

$$K_t i_a = K_f\frac{d\theta}{dt} + J\frac{d^2\theta}{dt^2} \qquad (8)$$

Laplace transform of the equation. assuming all initial condition equal zero, we have

$$K_t I_a(s) = s^2 J\theta(s) + sK_f\theta(s) \qquad (9)$$

$$KU(s) = R_a I_a(s) + L_a I_a(s) + K_b\theta(s) \qquad (10)$$

The effect of torque is as a result of the current flowing through the armature of the motor which by virtue of relating the current flowing to the applied voltage, we can analyze the effect of torque. The overall transfer function of the motor can be deduced by computing the ratio of the output variable, which in this case is the armature current (producing torque) to the input variable which is the input Voltage. The overall transfer function become

$$G(s) = \frac{I_a(s)}{U(s)} = \frac{K\left(sJ + K_f\right)}{s^2 J L_a + \left(JR_a + K_f L_a\right)s + \left(K_b K_t + K_f R_a\right)} \qquad (11)$$

## 2.2 Analysis of the Brushless DC Motor with MATLAB

**Representation in MATAB:**

In this paper, the dynamic modelling of the Brushless DC motor was implemented using the s-domain transform expression. A MATLAB m-file was written for the Overall transfer function derived in s-domain and substituting into it, the motor constant. The time and frequency analysis was carried out using the Voltage as the input and the current as the output.

**Simulation Result**

Presented in this paper are the two types of analysis carried out on the Brushless DC motor which are the Time-Domain analysis and the Frequency- Domain analysis. These two responses are analyzed in the modelled DC motor using MATLAB.

**Time Domain Responses**

Step and Impulse responses were used as test signals for the system so as to avoid infinite steady state error of the modelled second order system of the Brushless DC motor. The responses as plotted by MATLAB are shown below:
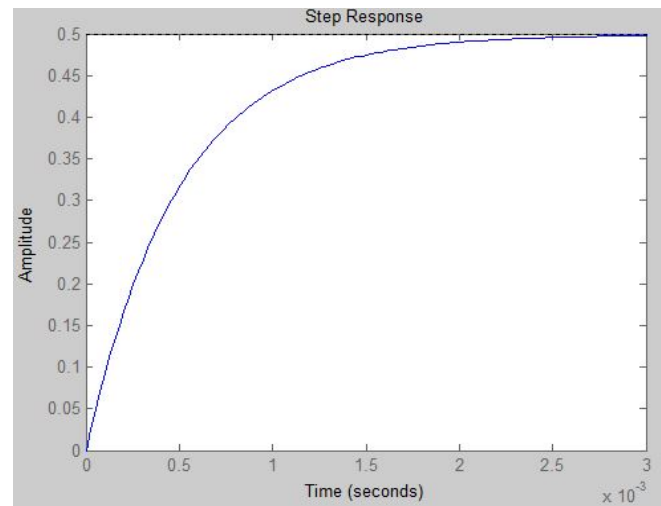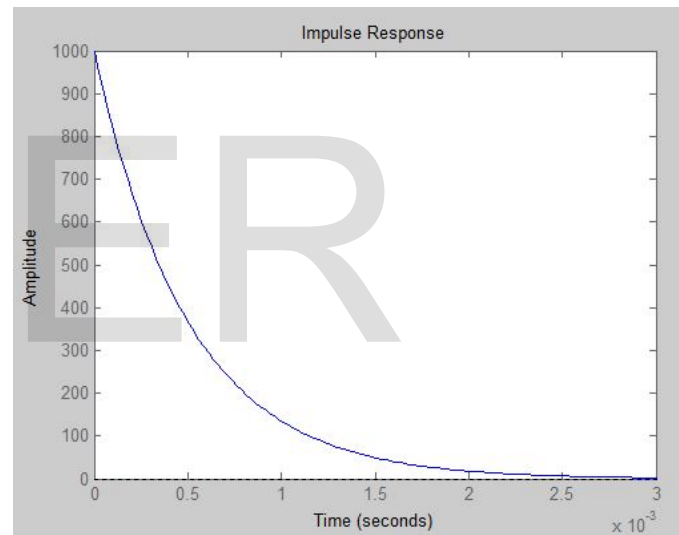


Fig 3 Step Response of the DC motor



Fig 4 Impulse Response of the DC motor

**Frequency-Domain Response**

For this Research, Bode Plot was used in analyzing the frequency response of the DC motor.
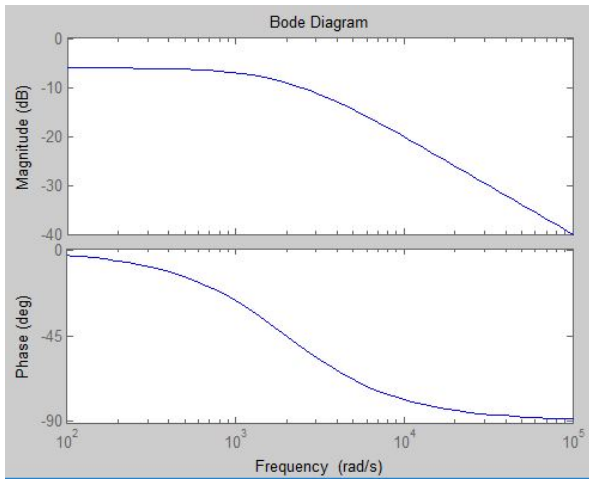
Fig 5 Bode plot of the DC motor

**Performance Evaluation**

The step response of the system shows a stable but sluggish system which there arise the need for designing a controller to adjust the time response of the system so as to achieve a system more stable with fast response and with little or no overshoot. PID controller is best as a controller to adjust the system to have these properties. The block diagram below shows a PID controller coupled to the system in a closed loop control.
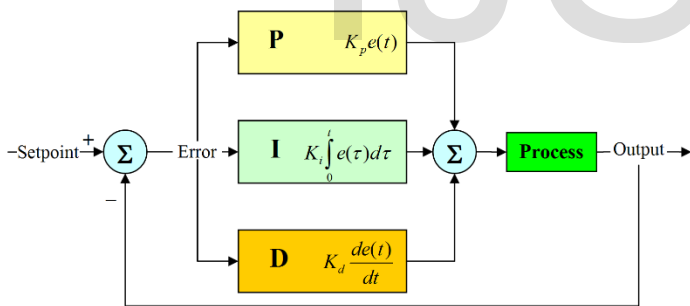


Fig 6 PID Controller connection to a system

By applying Laplace transform to the PID system, the transfer function of the system can be represented below with signal, u which is the output of the controller be the algebraic sum of the product of the proportional gain, Kp and the magnitude of the error, the product of the derivative gain, Kd and the magnitude of the error, and the product of the integral, $K_i$ with the magnitude of the error. Which mathematically is given by

$$u(t) = K_p + K_i \int e(t)dt + K_d \frac{d}{dt} e(t) \qquad (12)$$

And in s-domain it is represented by

$$U(s) = K_p + \frac{K_i}{s} E(s) + sK_d E(s) = \frac{K_d s^2 + K_p s E(s) + K_i E(s)}{s} \qquad (13)$$

Where U = controller output

E = controller input

Kp = proportional gain

Ki = integral gain

Kd = derivative gain

To get the value for the constant of the controller, Ziegler Nicholas algorithms was used and the step response of the system after the controller is added is shown below:
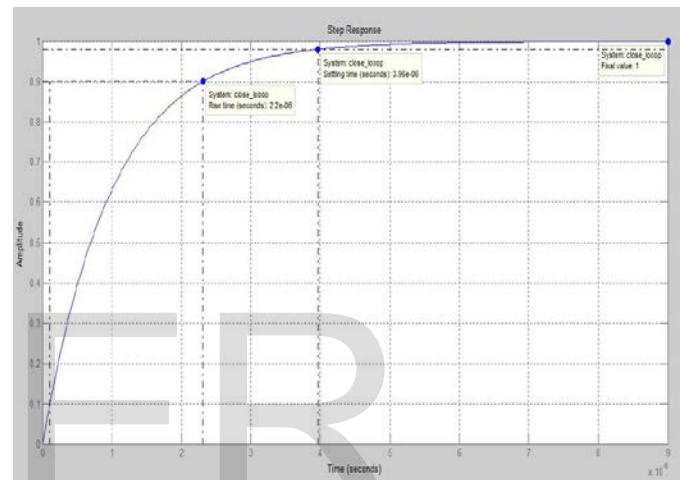


Fig 7 Step response of the System with PID Controller

## 2.3 Modelling Effect of Torque on Quadcopter Stability

The Torque generated as each motor rotates is given as

$$\tau_i = k\omega^2{}_i + I_m \omega_i \qquad (14)$$

Where

$\tau_i$ is torque created by each motor in Newton-meter (Nm), k is the drag constant $\omega$ is the angular velocity of the motor in radian-per-second (rad/s) and $I_m$ is the inertia moment of the rotor in kilogram-square meter (kg.m²) which is negligible most times due to the smaller value of its radius compared to its mass. At steady state where there seems to be no tale off or landing, the propeller maintains a constant or almost constant thrust and won't be accelerating. Hence the inertia is ignored and the equation for torque is simplified to

$$\tau_i = (-1)^{i-1} k\omega^2{}_i \qquad (15)$$

Where

the term $(-1)^{i-1}$ is positive for the ith term propeller, if the propeller is spinning in the clockwise direction an negative

otherwise. Hence sum of all the torque from each propeller can be written as

$$\tau_i = k(\omega^2{}_1 - \omega^2{}_2 + \omega^2{}_3 - \omega^2{}_4) \qquad (16)$$

Analysis of the stability of the quadcopter is necessary to check the behaviour and response of the quadcopter to a finite input signal. As it could be seen from above the system is a Second Order System and hence can be analyzed using the basic stability analysis tools. The Stability of the system requires proper analysis to avoid oscillations when excited. The whole quadcopter is therefore modelled by grouping similar motors producing sum-able torque together and analyzed as shown using block diagrams below;
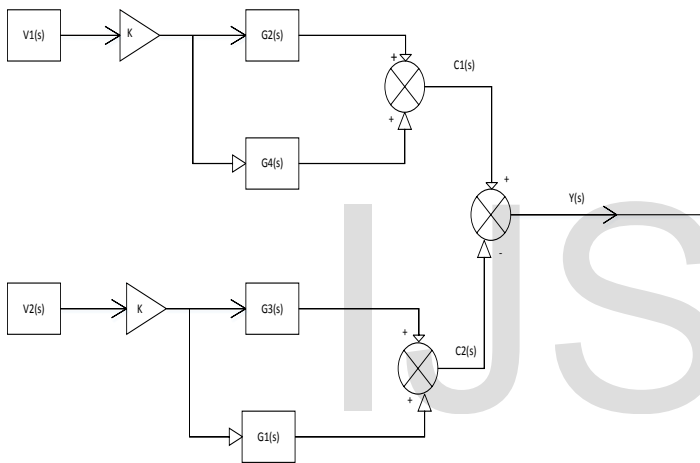


Fig 8 Block Diagram of a Quadcopter Model

For easy analysis, the transfer function of each of the motors on the quadcopter will be represented using $G_1(s)$, $G_2(s)$, $G_3(s)$, $G_4(s)$ for the four motors respectively. The transfer function of summers is given in equations below

$$\frac{C_1(s)}{V_1(s)} = G_1(s) + G_3(s) \qquad (17)$$

$$\frac{C_2(s)}{V_2(s)} = G_2(s) + G_4(s) \qquad (18)$$

And the overall transfer function is given by

$$Y(s) = C_1(s) - C_2(s) = (G_1(s) + G_3(s))V_1(s) - (G_2(s) + G_4(s))V_2(s)) \qquad (19)$$

Five different cases could be considered and the stability of the system at each instance checked.

**CASE 1:** $V_1(s) = V$ and $V_2(s) = 0$. The output equation becomes

$$Y(s) = C_1(s) = 2(G_1(s))V \;;\; G1(s) = G3(s) \qquad (20)$$

**CASE 2:** $V_2(s) = V$ and $V_1(s) = 0$. The output equation becomes

$$Y(s) = -C_2(s) = -2(G_2(s))V \;;\; G2(s) = G4(s) \qquad (21)$$

**CASE 3:** When $V_1(s) = V_2(s) = V$.

$$Y(s) = C_1(s) - C_2(s) = (G_1(s) + G_3(s))V - (G_2(s) + G_4(s))V = 0 \qquad (22)$$

The overall equation reduced to zero showing the system is at stable point

**CASE 4:** When $V_2(s) < V_1(s)$; say $V_2(s) = V$ and $V_1(s) = X\,V$. where X is the rational constant which can be varied from 1 to any number to observe the system behaviour.

$$Y(s) = C_1(s) - C_2(s) = (G_1(s) + G_3(s))XV - (G_2(s) + G_4(s))V \qquad (23)$$

**CASE 5:** When $V_2(s) > V_1(s)$; say $V_1(s) = V$ and $V_2(s) = X\,V$. where X is the rational constant which can be varied from 1 to any number to observe the system behaviour.

$$Y(s) = C_1(s) - C_2(s) = (G_1(s) + G_3(s))V - (G_2(s) + G_4(s))XV \qquad (24)$$

**EXPERIMENTAL SETUP**

The experimental setup for the research was designed and setup as shown below with the quadcopter hanged on a pivot



Fig 9 Quadcopter hanged to the Pivot

The experimental setup for the quadcopter consists of carbon fiber Frame, four Brushless DC Motor, four Electronics Speed Controller (ESC) which aid the switching on and off sequence of signal when the Pulse width modulated signal is sent, a Flight Control Board (Arduino Uno), Propellers attached to each of the brushless DC motor, Inertia Measurement Unit (MPU6050) for getting readings of the angular rotation of the quadcopter when operating, a power distribution board that

distribute power to the entity of the component on the board and a LIPO Battery (3 cells) to supply power to the circuit at a voltage level of 11.1V. the complete circuit arrangement is shown in figure below
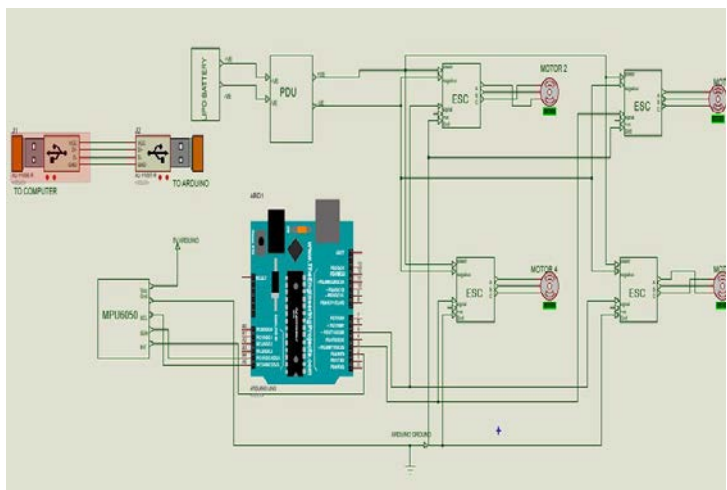


Fig 10 detail connection of the circuit

The input Voltage to the motors of the quadcopter rotating in same direction is adjusted in relation to others rotating in different direction and the effect of torque on the quadcopter physical behaviour is observed.

**RESULTS AND DISCUSSION**

Model of a brushless DC motor considering the effect of torque on its stability is used in determining its generalized equation which was further used in matlab for analysis of its time and frequency domain response. To achieve the modelling and simulation, the following steps was followed;

1. Circuit diagram of the brushless DC motor was represented
2. General equation of the system was created and its transfer function was determined
3. A model block was developed which was simulated in matlab to observe its performance
4. Four similar of such was grouped to model a typical quadcopter according to their direction of rotation.
5. Different cases were considered to simulated the effect of torque on the model.

Datasheet of the motor used, Emax Mt1806 2280Kv Brushless motor was looked into and some laboratory measurement were carried out in deriving its constant parameters. The value of the constant parameters of the motor computed is tabulated below and the motor datasheet is shown below,

Table 1 Motor constant

| S/N | Symbol | PARAMETERS | VALUE |
|-----|--------|------------|-------|
| 1 | J | Moment of Inertia (kg/m²) | $2.38 * 10^{-6}$ kg/m² |
| 2. | $K_b$ | Back EMF, (V-sec/rad) | $4.20 * 10^{-3}$ V-sec/rad |
| 3. | $K_t$ | Toque Constant, (Nm/A) | $4.20 * 10^{-3}$ Nm/A |
| 4. | $K_f$ | Coefficient of Friction, | $4.20 * 10^{-3}$ |
| 5. | R | Armature Resistance, (Ohms) | 1 Ohms |
| 6. | L | Armature Inductance, (H) | 0.001 H |
| 7. | K | Input Voltage constant, | 1 |

Substituting the values obtained from the table above into the final transfer function of the BLDC motor is given as.

$$G(s) = \frac{1000s + 1765*10^3}{s^2 + 2765s + 1773*10^3} \qquad (24)$$

The result of the system time domain parameters before and after the PID system was attached is tabulated below giving the PID constants parameters after tuning with Ziegler Nicholas algorithms to be

Kp = 1000, Ki = 1.9540e-06 and Kd = 4.8850e-07.

Table 2 Time response analysis parameter

| Response Type | Controller | Settling time | Rise Time | Over shoot | Peak time |
|---------------|-----------|---------------|-----------|-----------|-----------|
| Step | without controller | 0.0019s | 0.0011s | 0% | 0.0038s |
| Step | With controller | $3.9603*10^{-6}$s | $2.205*10^{-6}$s | 0% | $1.054*10^{-5}$ s |

### Simulation result of quadcopter model

Considering the cases highlighted above for varying input voltage to the motors of the quadcopter, Matlab m file was created to simulate the result of the mathematical model and it results are illustrated below;
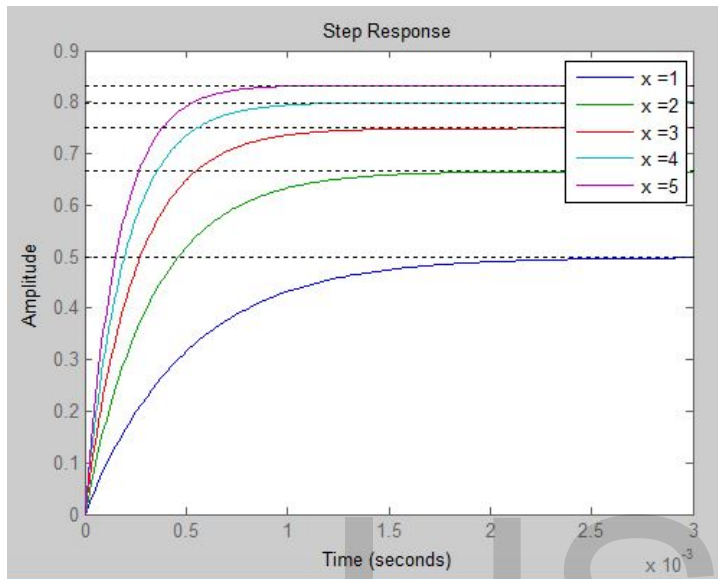


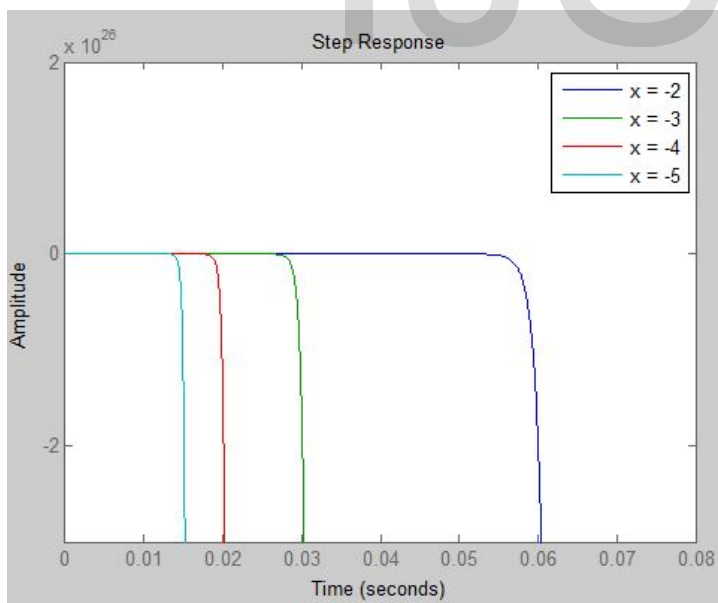Fig 11 Step response of the quadcopter for varying positive torque on the system



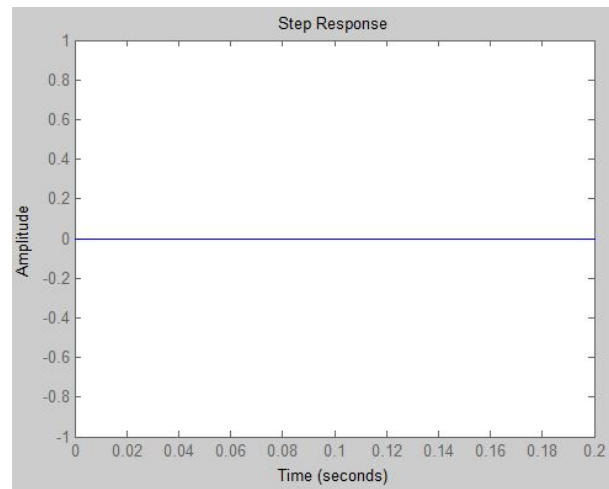Fig 12 Step response of the quadcopter for varying negative torque on the system



Fig 13 Step response of the quadcopter for equal torque on the system

## 3 CONCLUSION AND FUTURE WORK

The study of the quadcopter behaviour was carried out in this research by modelling the equation of a quadcopter considering the effect of torque as current flows through its motors and its transfer function derived. This made it easy to be able to use matlab in analysis it optimal performance. A PID controller was designed to adjust the stability of the Brushless DC motor used on the Quadcopter and evaluate its performance. Tuning this controller was achieved using Ziegler-Nicholas Algorithms which resulted in a fast system with no overshoot and a very minimal steady state error. A step signal was used to check the time response of the quadcopter system due to the system being a second order system and its steady state parameters were determined.

Future work to be carried out on this research includes by not limited to deriving the state space equation of the model thereby given room for carrying out the Controllability and Observability test on the model to see how controllable and observable the motor used on the quadcopter is. Better pivot can be designed which provide more degree of freedom movement for the quadcopter. Simulink Model of the quadcopter mathematical behaviour provided in this paper could be done and converted to code programmed on a FPGA chip which can be incorporated into the design of a quadcopter to tackle the effect of torque.

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